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Mathematics News Letter

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A Challenge to Forward-looking Mathematics Teachers in the Colleges and High Schools of Louisiana and Mississippi.

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VOL. 3 BATON ROUGE, LA., NOVEMBER, 1928 NO. 3

GREETINGS FROM PROFESSOR SLAUGHT TO LOUISIANA-MISSISSIPPI

It is with great pleasure that I observe the progress which is evident in the cooperation of the Mathematical Association of America and National Council of Teachers of Mathematics in Louisiana and Mississippi. I wish to congratulate all of you loyal supporters of the "News Letter", both contributors and subscribers. You are doing a good work and it is bound to produce an uplifting influence on the whole mathematical situation in that part of the country. I recall with great pleasure and satisfaction the two of your meetings which I have attended—Baton Rouge and Shreveport—where you made history, the former for the Association, the latter for the Council. I am sure it was an important step when you determined to **cooperate** in these two organizations. Some people in other parts of the country who are contemplating similar cooperation have raised the question whether one or the other group would not dominate the meetings to the detriment of the other. This might be the case if there were complete amalgamation of the two groups, but this is precisely what is not intended and I am sure you will not make such a mistake in Louisiana-Mississippi.

I hope that the Council Branch will always hold one session in which the interests of secondary mathematics shall have

primary consideration, that the Association Section will do likewise for collegiate mathematics, and that one joint session will be held where each group may have the opportunity to see the other in action and where each may learn to appreciate fully the problems confronting the other—in general, yes, but in particular in Louisiana and Mississippi. You both have problems all your own as well as those which concern the country as a whole and I am sure you will be inspired by your cooperative activities to do all in your power for the betterment of mathematical teaching and for the promotion of mathematical appreciation in your own communities.

You will have seen by the October issue of the **Mathematics Teacher** that the National Council is now an incorporated body and is in a position of strength to move forward more actively than ever before. The organization of new Branches is going forward with acceleration and everything is all ready for the campaign to put the Council over the top with ten thousand members by 1930. You in Louisiana and Mississippi can do much to help toward this end by assisting teachers to realize that the **Mathematics Teacher** is indispensable for everyone who is teaching high school mathematics. There is to be a Register of members printed this fall and it will be a veritable "Who's Who" for secondary mathematics. Professor W. D. Reeve of Teachers College, Columbia University is now in full editorial charge. This ensures that the magazine will be enlarged and enriched just as rapidly as the increasing receipts from memberships will warrant.

With cordial greetings to all,
H. E. SLAUGHT

TO MATHEMATICS NEWS LETTER SUPPORTERS

1. Since the News Letter was placed on a subscription basis, more than a year ago, only one hundred sixty-nine paid-up subscriptions have been received by the News Letter management.

2. There are more than one thousand teachers of mathematics in Louisiana and Mississippi who have not as yet subscribed to the Letter.

3. Five hundred copies of the Letter were printed in each

of the months, September and October, of this year, practically all of which were mailed out to mathematics teachers of the two States.

4. This means that over three hundred copies were sent out each of these months to non-subscribers.

5. The policy of mailing out the Letter to a fairly large number of non-subscribing mathematics teachers was carefully considered in a conference between the Editors and Secretary Cole. As a result of the conference it was decided that the influence of the News Letter would be materially curtailed if its distribution should be confined merely to those whose names are on our subscription list.

6. A few have been sufficiently interested to inquire of us why the News Letter continues to be published in and issued from Baton Rouge, contrary to the resolution passed by the Louisiana-Mississippi Section and Council at their joint annual session held in Jackson, Mississippi, last March, namely, that in the future the News Letter should be printed by the L. P. I. printing plant at Ruston, Louisiana, in view of an offer of the L. P. I. administration to print the Letter at a certain low cost.

We tender the following answer to such inquiries: Within a short time after the joint session of Section and Council had accepted the above offer from the then existing administration of L. P. I,, a new administration took charge and the new administration notified the News Letter management that it was unwilling, for the present at least, to assume responsibility for the printing of the Letter at Ruston.

7. Since the Jackson Section-Council had resolved that eight issues of the Letter are to be put out each year, the management, promptly after the above notification, sought in Baton Rouge a printing firm to handle the News Letter, one which would do so as inexpensively and efficiently as possible.

8. The Ortlieb Press, of Baton Rouge, has contracted to furnish a twenty-page journal each month at the rate of two dollars per page for the first two hundred copies. For each additional hundred copies printed, the additional cost is to be one dollar and a half. Thus the cost of five hundred copies of the Letter is ($44.50) forty-four dollars and a half.

9. It costs ($8.24) eight dollars and twenty-four cents to

mail out five hundred copies of the Letter at third-class postal rates.

10. On account of the L. T. A. meeting in Baton Rouge 1,000 copies of the November issue will be printed.

11. There shall be no News Letter debt to be reported to the Section-Council session at its next annual meeting to be held in Lafayette, Louisiana.

MEMORANDA

The capacity of the average mind for being wholly guided by other minds diminishes with the passing of time. It is easier for the junior high teacher to permanently correct student errors in the fundamental operations of arithmetic than it will be later for the senior high to correct them. It will be easier for the senior high instructor to correct them than it will be a year later for the college freshman teacher to do so, but easier for the latter than it will be for the sophomore instructor to do it after twelve more months shall have gone by. Indeed is he, or is he not, mathematically hopeless who reaches the second year of his college period still nursing the delusion that $(a/b+b/d)=a/d$?

A new mathematics book, like a new human being, a young man, for example, is apt to be guilty of some errors. The errors of a text are the opportunities of a competent teacher. But he must be competent.

A one-hundred per cent response to the obligations of one's teaching profession implies one's eager—yes, watchful—readiness to seize every proffered means to increase teaching effectiveness. Are the students given too little work to do? Are they given too little? Are the discussions clear? Is too much, or too little, explanation being offered? Is the spark of the student's individual interest in his course work being properly developed? Answers to all these questions, whether favorable or unfavorable to one's teaching, are alike of use to the progressive teacher. If unfavorable, as far as may be necessary he will re-orient his method, or attitude, or both, or will even reform his teaching. If favorable, he will be properly stimulated to even better work,

taking care to avoid the evil of over-complacency, maintaining a critical or skeptical attitude to any pronouncement that his teaching cannot take on further improvement. —*S. T. S.*

SUCCESSFUL MATHEMATICIANS—JAMES STERLING

HARRY GWINNER
University of Maryland

James Sterling, Mathematician, Inventor and Manager was born in Scotland in 1692. He was the son of Archibald Sterling and Anna, eldest daughter of Sir Alexander Hamilton, there being four sons and five daughters.

He studied for a time at Glasgow (?) University and entered Balliol College, Oxford University, England, in 1711. He was a very diligent student, or he could never have acquired the scholarship that bore fruit in 1717 in the production of his **Lineae Tertii Ordinis,** a work which is still a recognized commentary on Newton's **'Enumeration of Curves of the Third Order.'** While at Oxford he was not behind in the bold expression of his political and religious opinions, and he took a leading part amongst Balliol students in the disturbances of 1714-1716 and he is specially mentioned in the List of Persons concerned in the Rebellion of 1745-46.

In 1715 Sterling first gave indications of his ability as a mathematician. On Feburary 24, 1715, John Keill, of Oxford, in a letter to Newton, mentions that the problem of **orthogonal trajectories,** which had been proposed by Liebniz, had recently been solved by 'Mr. Sterling, an undergraduate here', as well as by others.

The theorem

$$y=f(x)=f(o)+f'(o)\frac{x}{1}+f''(o)\frac{x^2}{2!}+f'''(o)\frac{x^3}{3!}+\ldots$$ is a theorem by which a function of a single variable may be developed into a series of terms arranged acording to the ascending integral powers of that variable, with constant coefficients. It is generally known as Maclaurin's, having been first published by him in 1742; but as it had previously been given in 1717 by Sterling, it is credited to Sterling—the subject of this sketch.

In 1717, he was offered a Professorship of Mathematics in Italy—very likely Padua. He was compelled to refuse the proffered chair, due to some religious difficulty. Although he did not occupy the chair, he lived for some time in Venice. He was heard of again in 1924 when he was making an effort to get into business.

In 1725 he acquired an interest in Watt's Academy, London, where he taught mechanics and experimental philosophy. While at the Academy, his studies were directed towards the problem of the figure of the earth, the discussion of which had given rise to two rival theories, (1), that of Newton, who maintained that the Earth was flatter at the Poles than at the Equator, and (2), that of Cassinis, who held the opposite view.

In 1735, a great change in his circumstances was occasioned by his appointment to the managership of the Leadhill Mines in Scotland. While in Venice, he had, at the request of certain London merchants, acquired information regarding the manufacture of plate glass. Indeed, it is asserted by some that owing to this discovery he had to flee from Venice as his life was in danger. When Sterling was appointed manager of the Company the affairs were in a bad way.

In letters to Maclaurin (the mathematician), he complained he had no time to devote to scientific researches and when writing to Euler (one of the greatest mathematicians who ever lived) he tells him that he is so engrossed in business that he finds difficulty in concentrating his thoughts on mathematical subjects in the little time at his disposal.

Gradually the debts which had accumulated in his predecessor's day were cleared off, and the mines became a source of profit to the share-holders. The village in which he and the miners lived was a bleak spot in bare moorland, nearly 1300 feet above the sea level. There was no road to it, and hardly even a track. In spite of these disadvantages Sterling has left indelible traces of his wise management, and many of his improvements have a wonderful smack of modernity. The miners were a rough, dissipated set of men, who had good wages but few of the comforts of life. Sterling's first care was to add to their comfort and to lead them by wise regulations to advance their own physical and mental welfare.

On the other hand, Sterling's own requirements were well

provided for by the Company, whose affairs were so prosperous under his control. They saw to it that he was well housed, while the salary they paid him enabled him to amass a considerable competency. For thirty-five years Sterling held his managership.

Sterling married a Miss Barbara Watson, their only child being a daughter, who married her cousin, Archibald Sterling, and who succeeded her father as manager of the mines. He died in 1770, and was buried in Greyfriars Churchyard.

He invented Sterling's Engine, which was designed to create a current of air, due to falling water, sufficiently strong to blow a forge or to supply fresh air in a mine.

Published Works

Commentary on Newton's "Enumeration of Cubics." In this, Sterling proved all of the theorems of Newton up to, and including, the enumeration of cubics.

The Methodus Differentialis. This is not a treatise on the Differential Calculus, but is concerned with the Calculus of Finite Differences.

Sterling's influence as a mathematician of profound analytical skill has been a notable fact within the inner circle of mathematicians. Witness, for example, the tribute of praise rendered by Laplace in his papers on Probability and on the Laws of Functions of very large numbers. Binet, in a celebrated memoir on Definite Integrals, has shown Sterling's place as a pioneer of Gauss. Gauss himself had most unwillingly to make use of Sterling's Series, though its lack of convergence was anathema to him.

This account of Sterling's life is from Charles Tweedies' volume "James Sterling", published in 1922 by the Clarendon Press, Oxford, England.

ON THE PROBLEM OF INTEGRATION BY PARTS

C. D. SMITH
Louisiana College

The object of this note is to give a brief disscusion of a paradox mentioned recently by J. L. Walsh[1] which results from the use of the formula

(1) $\int udv = vu - \int vdu.$

In applying (1) to the function $udv = dx/x$ the result is

$$\int dx/x = 1 + \int dx/x$$

and the method not only fails but yields the absurdity $0 = 1$. Unfortunately the equation appears in form (1) in many introductory texts on the calculus without mention being made of necessary restrictions. The result is that the beginner who meets a situation like the above is often left in confusion. Let us try to find what is wrong with equation (1).

Consider the differential equation

$$d(uv) = udv + vdu,$$

or

$$udv = d(uv) - vdu.$$

Integrating we get the formula

(2) $\int udv + c = uv - \int vdu$

which contains the constant of integration as is the case in general with the indefinite integral. Now, just as is the case with all other integration formulas, it is true of (2) that it does not apply in general altho the equality does hold. The method obviously fails to find $\int udv$ whenever

(3) $\int udv + \int vdu = 0.$

If we apply (2) to the integral $\int dx/x$ we arrive at the result $c = 1$, thus avoiding the paradox but failing at the same time to find the integral since condition (3) holds for $u = 1/x$, $dv = dx$.

It seems that one might suggest in conclusion that text books intended for a first course might do well to develop equation (2) and add restriction (3) when introducing the method of integration by parts. The beginner would then have a correct formula with the limitations indicated and he would not be left open to the confusion which often results from the assumption that a mathematical formula is true in general when such is not the case.

*1. The American Mathematical Monthly, Vol. xxxiv, No. 2, Feb. 1927.

NOTE ON PARAMETRIC EQUATIONS OF THE STRAIGHT LINE

By H. L. SMITH
Louisiana State University

In this note a set of parametric equations of the straight

line will be derived by means of the fact that the straight line segment joining two points is the shortest path connecting them.

1. Linear coordinate system. Let P_0, P_1 be two fixed points of a given line, and let P be an arbitrary point of the line. Then to P we assign the coordinate t defiined as follows:

$t=\overline{PP_0}/\overline{P_1P_0}$, if P_0 is not between P_1 and P;

$t=-\overline{PP_0}/\overline{P_1P_0}$, if P_0 is between P_1 and P.

It is clear that P_0, P_1 have coordinates 0 and 1 respectively, and that the coordinate system is uniquely determined by this fact; we say that t as above defined is the coordinate of P in the system determined (on P_0P_1) by P_0,P_1 (as origin and unit-point, respectively).

2. Theorem from plane geometry. We assume, for use in the next section, the following theorem from plane geometry:

THEOREM. *If A, B, C are three points and* $\overline{AC}=\overline{AB}+\overline{BC}$, *then B is on the segment joining A and C.*

3. Derivation of the parametric equations. Let $P_0(x_0y_0)$, $P_1(x_1y_1)$ be two points. Set

(1) $\quad h=x_1-x_0,\ k=y_1-y_0$

so that $x_1=x_0+h$, $y_1=y_0+k$. Let t be any real number and consider the point P (xy) where

(2) $\quad x=x_0+ht,\ y=y_0+kt.$

By the distance formula and (1), (2),

(3) $\quad \overline{P_1P_0}=\sqrt{h^2+k^2}$

(4) $\quad \begin{cases}\overline{PP_0}=\sqrt{h^2t^2+k^2t^2}=\sqrt{t^2(h^2+k^2)}=|t|\overline{P_1P_0}\\ \overline{PP_1}=\sqrt{(t-1)^2(h^2+k^2)}=|t-1|\ \overline{P_1P_0}.\end{cases}$

Now suppose that $0<t<1$. Then $|t|=t$, $|t-1|=1-t$ and (4) become

(5) $\quad \overline{PP_0}=t\ \overline{P_1P_0},\ \overline{PP_1}=(1-t)\overline{P_1P_0}$

so that

(6) $\quad \overline{P_0P}+\overline{PP_1}=\overline{P_0P_1}.$

By § 2, it follows from (6) that P is on the segment joining P_0 and P_1, and it then follows from (5) (by § 1) that t is the coordinate of P in the system determined by P_0, P_1.

Next suppose that $1<t$. Then $|t|=t$, $|t-1|=t-1$ and (4) become

(7) $\quad \overline{PP_0}=t\ \overline{P_1P_0},\ \overline{PP_1}=(t-1)\overline{P_1P_0}$

so that

$$\overline{PP_0}-\overline{PP_1}=\overline{P_1P_0}$$

or (8) $\overline{PP_0}=\overline{PP_1}+\overline{P_1P_0}$

It now follows (§ 2) from (8) that P_1 is on the segment joining P and P_0 and it then follows from (7) by § 1 that t is the coordinate of P in the system determined by P_0, P_1.

Next suppose $t<0$. Then $|t|=-t$, $|t-1|=1-t$ and (4) become

(9) $\overline{PP_0}=-t\ \overline{P_1P_0}$, $\overline{PP_1}=(1-t)\overline{P_1P_0}$

so that

$$\overline{PP_1}-\overline{PP_0}=\overline{P_1P_0}$$

or (10) $\overline{PP_1}=\overline{PP_0}+\overline{P_0P_1}$.

It now follows from (10) that P_0 is on the segment joining P to P_1 and then from (9) that t is the coordinate of P in the coordinate system determined by P_0, P_1.

Finally if $t=0$ or $t=1$ then $P=P_0$ or $P=P_1$ and a similar conclusion follows. We have thus proved the

THEOREM. *If* $P_0(x_0y_0)$, $P_1(x_1y_1)$ *are two points, then the point* $P(xy)$, *where*

(11) $x=x_0+(x_1-x_0)t$, $y=y_0+(y_1-y_0)t$,

is the point of P_0P_1 *whose coordinate is* t *in the coordinate system determined by* P_0, P_1.

Equations (11) are said to be parametric equations of line P_0P_1.

Applications. Equations (11) may be made the basis of the entire theory of the straight line. We note also that the formulas for the mid-point of a segment are given by (11) for $t=\frac{1}{2}$, and the formulas for the point which divides a given segment in the ratio k_0: k_1 is given for $t=k_0/(k_1+k_1)$.

SUMMARY OF PRESIDENT HARRY C. BARBER'S ADDRESS ON MATHEMATICS IN THE JUNIOR AND SENIOR HIGH SCHOOL AS DELIVERED TO MASSACHUSETTS TEACHERS

By ONE PRESENT

The key to Mr. Barber's talk on elementary mathematics can be found in a sentence near the close in which he refers to the fight which is being waged between the "drill master" and

the teacher who "would present the human significance of the subject." Elementary mathematics as it used to be taught in the senior high school was so much drill. The ordinary old text book was written so as to make any other treatment almost impossible. No account of the "fine art of teaching" was taken in these texts. There was no "letting a big idea grow slowly in the mind of the pupil; inserting one or two basic preliminary notions and setting them to work; by and by taking another step, adding a new notion and again using them all; and, a week or a month or a term later, the ripening and harvesting of the crop. No mathematics book is a good one which does not provide for such development of its main ideas. If you want an epigram: "Ideas are seeds to be planted and nourished; not pills to be swallowed at one gulp."

Forty years ago Mr. George W. Evans introdued into elementary algebra the study of the graph. With this was made a real beginning in the treatment of algebra as a symbolism for general ideas. We have been faced with a situation in which the symbol had been treated as more important than the idea, in which, in fact, algebra and geometry had largely become a study of mere "empty symbols." Preparation for an examination was made the end of our teaching of elementary mathematics. We lost sight completely of the exploratory idea which should be behind it. Algebra got cluttered with material of no practical value whatsoever and of considerable philosophic difficulty; geometry was fettered with an antiquated system of verbal algebra, the very use of which would prevent all application of really worth-while algebra to a subject in which it should be a primary aid. On the other hand, topics that are of primary interest and practical value were entirely omitted; for example, the development of a "number sense" and the ability to make rapid use of the many tables at our disposal.

Mr. Barber has devoted his life work to trying to bring about the needed reforms, and in his talk offered some specific suggestions to this end. Most of them are of interest chiefly to the teachers of mathematics to whom they were addressed. But behind them all is the single idea of substituting "understanding for mere mechanical manipulation." Reform had to begin in the Junior High School, and might reasonably be expected to begin there; the junior high school had a clean slate; it did not

have to contend with the obstacles of prejudice and inertia found in the senior high. Reform in the senior high school had to follow and must continue.

In the junior high school the pupil is taken in hand, and there in his mathematics the exploration of his mind and of the world about him is begun. The work that is given him is in line with his own experience. When he needs a new symbol, a system of short-hand writing, it is given to him, and then it is **used.** There is, for example, no mention of the miscalled "four fundamental processes". The pupil meets, learns to use, and understands the equation, the formula, the graph, and the geometric figure. They come to him as something helpful, practical, definite, as aids to his thinking and his interpretation. But when he entered grade nine, the continuity suffered a break; he was faced with a complete readjustment which took away from him most of the interest of the subject and substituted for it a series of mechanical rules. He began "formal" algebra and geometry.

Instead of this, Mr. Barber contends that while the work of the ninth year may be mainly devoted to algebra, there should be correlation of this algebra with geometry. Quite aside from the vastly increased interest for the pupil, there is the practical value resulting from the new source of problem material thus made available and from the great reduction of difficulty which the pupil ordinarily experiences from the formality with which his geometry is now first presented to him. "I think," Mr. Barber said, "I can reduce the difficulty of geometry twenty-five or thirty per cent by proper use of geometric material in the algebra class, and incidentally make the algebra rather more interesting and valuable than less."

It is into this ninth year also that the trigonometry unit which has found a place in our algebra should be introduced. It affords a good illustration of algebra applied to geometric material, an illustration far more important, valuable, interesting and simple than much of the old formal algebra which is being replaced.

In the tenth grade the pupil makes his deeper study of geometry, and the teacher's aims must be to keep algebra alive by the its further application to this geometry. The axiomatic and algebra are doing away with most if not all of the verbal algebra with which geometry has been so badly encumbered.

The use of algebraic proofs whenever they are possible makes the pupil realize more fully the value of this algebra and its place in his later mathematics.

If an idea is important, it has its use. If it is important, it should be developed early in the course and then used over and over again in its many applications. Only in this way will the pupil keep thinking about these important ideas, will he realize their value and their application to this scientific age in which he lives, and will he keep growing. When this is done, mathematics will take its proper place in our high school curricula.

It might not be without significance to notice that Mr. Barber also asserted that the teacher who tried conscientiously to teach mathematics in the new way, would find herself far more interested in her task and far more convinced of its great and lasting importance.

Attention was called to the Massachusetts State Course of Study in Mathematics as a document worthy of careful and repeated reading.

Why should the Mathematics News Letter be confined to two states? Twenty-five subscribers from each of forty-eight states would mean twelve hundred dollars a year in subscription money.

PROGRAM OF MATHEMATICS SECTION OF LOUISIANA TEACHERS' ASSOCIATION

(Baton Rouge, La., Feb. 22-24)

The Minimum Training Prerequisite to the Successful Pursuit of College Mathematics.....Prof. A. C. Maddox, State Normal

Principles and Aims of the La.-Miss. Branch of the National Council of Mathematics Teachers .....Mr. Henry Schroeder, Ruston High School

The Value of Solid Geometry, and Its Rightful High School Mathematics Course .....Principal Grover C. Koffman, Shreveport High School

THE TEACHING OF ALGEBRA CAN BE MADE INTERESTING

By DAISY WHITE
Ruston High School

It is a pleasure to teach mathematics, especially algebra, to a class of beginners. Every new class is an adventure for the teacher as well as for the class. Together they explore, sometimes gropingly, sometimes enthusiastically, into the mysteries of the unkown. The teacher needs must be the captain of this voyage and very largely responsible for the success and enjoyment of all who undertake this tour of discovery.

The unkown to be worked for and cherished when discovered is the secret formula for arousing and sustaining interested attention. The first requisite, a sine qua non, is the interest of the teacher herself. How can a teacher lack interest when eager inquiring minds await inspiration and guidance!

The majority of those beginning the study of algebra really like and enjoy the work. The antiquity of the science is a big thought. Some wide awake boy or girl may raise the question as to whether there is anything in life to be discovered or worked out in this realm. The conclusion of the discussion that naturally would follow would be that new applications may be made if no new laws can be discovered. Biographies of great scientists, inventors, engineers, manufacturers, explorers, ancient and modern, are gladly read and reported upon, especial interest being taken in the part mathematics has played in their success. Other bits of history, such as the origin of number systems, origin of the various signs and symbols, the history of fractions, the history of pi, etc., add to the interest of the study.

The class can be led to appreciate the importance of practice and drill. They know that a skilled workman must not only know how to use his tools but when to use the different kinds to best advantage and good results. A lively and interesting period of the recitation can be spent in making formulas, "algebraic declarative sentences", for the same fact, rule or law, and then selecting the best from the many given. Translation from words to symbols and from symbols to words, practice in substition or evaluation of algebraic expressions, are exercises that

can be used frequently, gradually advancing from the easy to the more difficult. Equations cannot be stressed too much nor too much practice given in them. Performing mechanical operations in algebra and arriving at the "right answer" affords a good pastime but the pupils who prefer this to problem solving miss the best thrills.

Practical problems, concrete problems, real problems, suitable problems, are hard to find or to make. The children like to compose problems. If you have exhausted your supply of good problems try this method of replenishing them. Some of the best problems I have used have been made by the members of the class. One of our leading educators told me that he had his class to make a text book in algebra and they had great fun doing so and made a good job of it.

Some answer, carefully given and as satisfactory as possible, must be made to the question, "what use will anybody have for algebra?" From the experiences of older brothers and sisters they can learn that a knowledge of algebra is necessary to their success in science and other mathematics in their further work in high school and in college. If their future vocation or profession has not been decided upon it might happen that algebra will prove very useful in the choice they will make.

To insure the success of this venture we must remember the old much emphasized essential. A clear, definite, full assignment will do more to insure a successful preparation and a profitable recitation than, perhaps, any other one thing.

We have this problem ever before us. How can we make our assignments invigorating and stimulating? How can we present the subject matter so that the boys and girls will progress with "ease, interest and thoroughness?"

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*TYPES OF THINKING IN ALGEBRA

By CORA DROZ BROUSSARD
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Interpretation of Tables

The fact that very little difference was found to exist between boys and girls would seem to disprove the theory that boys are more proficient in algebra or mathematics in general.

Boys at the end of six months training are more able to solve simultaneous questions. The percentage of errors for the girls was 44.9% while that of the boys was 25%.

The table representing the per cent of errors in relation to months of training is fairly consistent throughout. The percentages of errors, as a whole, decrease when the amount of training is increased. The greatest number of cases fall into the six months group thus making it more of a measure than any of the others. The five and the five and one-half month groups contain comparatively few cases. This explains, to some extent, the cause of the increase in the percent of errors at this point. Another idiosyncrasy of the same kind is that of the seven months group in which there are only ten cases. These pupils made the average percent of errors high because they were very weak in all phases of the subject.

It is interesting to note that a large percent of errors in subtraction continues to exist after fifteen months of training. Table I shows that 34.5 percent of errors are made at end of six months of training. At eleven months 13.05% of errors are made; and at the end of fifteen months 10.45%. Most of these errors consist of the failure to apply the law of signs in subtraction; yet there are a few who didn't perform arithmetic accurately.

The percent of errors occur in the following order beginning with the highest: fractional equations, 46.32, formulae, 45.51, quadratic equations, 39.03, simultaneous equations, 36.14, substitution, 35.36, thought problems, 31.11, radicals, 28.01, subtraction, 22.86, factoring, 22.06, etc.

The least number of errors occurred in division although there is a tendency to forget the division processes when they are implicated in other problems. Pupils are able also to clear fractions when they are called on to do so; but when this process occurs in other examples, they fail to perform it accurately. This shows that there is need for its application while the children are learning to clear fractions. Perhaps drill and applications, more drill and still more applications are necessary to root the process.

Comparatively few errors were made in collecting terms, which indicates that this type of process is readily mastered.

Children fail to manipulate exponents efficiently. The de-

crease in the percentage of errors, indicated in table I, is gradual. At the end of eleven months training the percentage is 5.1 and at the end of fifteen months it is 12.45. The children who had studied algebra eleven months made a better rating because the subject was taught continuously instead of allowing a year to intervene as in the case of the fifteen months pupils. This would seem to be an argument in favor of teaching algebra two consecutive years rather than allowing Geometry to take its place in the third year.

Substitution is another bugbear to algebra pupils. The following evidence indicates that it is too concrete. Pupils are so used to the abstract in algebra that they can't convince themselves that anything concrete should be found in it. They insist on leaving letters after their values have been substituted. It wouldn't be algebra if there weren't leters! A remedy for this would be to impress the children with the fact, as Thorndike says, that the letters are real numbers.

In solving fractional equations the chief failure was in clearing fractions correctly, and strange to say, as indicated above, they performed the same process correctly when asked to do so. This would indicate that the mechanical processes are more easily forgotten than those that have reasoning in them. The same condition exists in parentheses.

The "mean" percentage of errors in the solution of formulae is very high, 45.51. Why are there so many errors in this type of problem? Perhaps this kind of equation is so unusual that pupils are unable to adjust their knowledge of other processes to examples of this sort. If we should make them mean something to pupils, by explaining their symbolism, we should make one step toward an easier solution. At present there is a tendency to treat this topic lightly.

The percentages of errors in reasoning problems are very high but not as high as was expected, for teachers are usually under the impression that more mistakes are made here than in any other algebraic process, but, according to table III, the "mean" percentage ranks sixth in amount of errors.

Table II represents the percentage of errors in relation to chronological age. The greatest percentage of errors occurs here as in table I in fractional equations and the lowest in multiplication, rather than in division as in Table I. Fewer errors

are made among pupils of from eighteen years to eighteen years and eleven months. Children of these ages have as a rule had about fifteen months training. Then again their maturity affects the quality of their work. In some cases, though, younger pupils made fewer errors, showing that age does not always influence the degree of achievement.

Table III is a tabulation of the percentage of errors in relation to the "Index of Brightness." The percentage of errors decreased as the I. B. increased. The greatest amount of errors was found in solving formulae rather than in fractional equations, as Tables I and II indicate. As in Table II, the least percentage of errors is in multiplication.

As a summary of the points brought out in this paper in discussing the types of algebraic thought we may formulate answers to the questions stated in the beginning.

The order of difficulty in the mastery of the various types of algebraic technique is: fractional equations, formulae, quadratic equations, simultaneous equations, substitution, reasoning, radicals, subtraction, factoring, simple equations, exponents, parentheses, expansion, collecting terms, clearing fractions, multiplication, and division.

The puipls' method of reasoning seems to be more or less fixed before they begin the study of algebra. If the teacher's line of reason had been followed fewer errors would be the result.

This study has a logic of its own in that the child has to change his method of reasoning to a great extent in order to adjust himself to this atmosphere. When he is unable to adjust himself, he is overcome by the complexity of the processes and thus fails to accomplish much.

Pupils vary to a marked degree in learning algebra. The degree of progress may be divided into three groups—fast, average, slow. Those in the fast group use clear rapid thinking, while those in the middle think more slowly and perhaps less clearly than the fast ones. The slow group consists of those who have not learned how to think clearly, or perhaps who cannot or will not do so. The teacher's problem here is to raise the standard of these groups.

*The first installment of this paper was published in the October News Letter—Editors.

PERCENT OF ERRORS IN RELATION TO MONTHS OF TRAINING

Months of Training	Collecting Terms	Substraction	Multiplication	Division	Expansion	Exponents	Factoring	Clearing Fractions	Substitution	Simple Equations	Fractional Equations	Formular	Parentheses	Quadratics	Radicals	Simultaneous Equations	Thought Problems
5 mo.	15.85	17.45		.9	20.7	21.45	27.3	12.8	39.2	19.4	49.95	49.1	16.6	45.8			32.
5½ mo.	34.95	26.6			23.3	16.6	33.3	23.3	50.0	15.0	35.00	54.85	22.50	35.0			32.65
6 mo.	13.6	34.5	2.65	1.2	17.35	23.85	14.75	10.35	30.1	11.75	50.4	48.25	15.65	48.65		34.95	23.45
7 mo.	22.8	35.65	2.85		28.3	35.65	35.65	17.35	52.85	39.25	68.55	48.55	27.85	53.8	30.97	44.5	46.25
11 mo.	6.25	13.05			4.95	5.1	5.1	5.6	21.75	13.2	33.7	30.0	9.35	30.2	34.75	31.6	28.2
15 mo.	6.65	10.45			7.00	12.95	16.25	5.7	18.3	2.55	40.35	42.2	16.65	20.7	18.3	33.50	22.35
Mean:—	16.28	22.86	2.75	1.05	16.93	19.27	22.06	12.52	35.36	20.16	46.32	45.51	17.26	39.03	28.01	36.14	31.11

PERCENT OF ERRORS IN RELATION TO CHRONOLOGICAL AGE

Chronological Age	Collecting Terms	Substraction	Multiplication	Division	Expansion	Exponents	Factoring	Clearing Fractions	Substitution	Simple Equations	Fractional Equations	Formular	Parentheses	Quadratics	Radicals	Simultaneous Equations	Thought Problems
13-13-11	.54	.44	.18	.13	2.1	.69	1.25	1.28	1.8	1.25	4.49	4.95	1.65	4.15	.7	2.95	2.75
14-14-11	2.6	2.2	.25	.35	2.9	4.35	2.9	2.0	5.45	2.6	7.94	8.29	2.35	4.85	.3	5.35	5.35
15-15-11	2.9	3.2		.65	3.05	3.3	3.85	1.0	7.55	.78	10.45	7.85	4.3	5.64	3.5	5.67	5.75
16-16-11	2.3	3.4	.18	.1	2.2	3.45	3.6	2.35	6.45	2.7	9.15	8.80	1.63	4.5	3.4	4.75	5.6
17-17-11	3.85	4.6	.29	.18	1.63	3.79	2.75	2.8	5.65	2.95	10.95	7.55	4.0	4.85	2.57	5.55	5.5
18-18-11	.95	1.59			.74	.9	1.95	1.8	2.35	.18	1.7	1.65	1.3	1.5	2.19	2.14	1.6
Mean:—	2.19	2.57	.22	.29	2.11	2.75	2.22	2.07	4.83	1.74	7.45	6.52	2.54	4.19	2.09	4.40	4.43

PERCENT OF ERRORS IN RELATION TO INDEX OF BRIGHTNESS

Index of Brightness	Collecting Terms	Substraction	Multiplication	Division	Expansion	Exponents	Factoring	Clearing Fractions	Substitution	Simple Equations	Fractional Equations	Formular	Parentheses	Quadratics	Radicals	Simultaneous Equations	Thought Problems
Below 40		.4			.15						.55	.55			.10		.55
41- 50	1.17	1.17			.8	.65	1.05	1	2.55	1.56	3.00	1.05	1.32	1.00	1.1	2.08	1.75
61- 60	.8	.55			1.57	.75	.75	1.17	1.57	1.57	3.15	1.57	.55	.25	.4	1.77	1.41
61- 70	2.79	1.1		.15	1.1	.8	2.3	2.08	3.48	1.36	4.25	4.5	.81	1.85	1.25	2.7	2.03
71- 80	2.15	2.87	.4	.15	3.00	1.97	2.4	2.28	3.54	1.87	5.9	4.64	2.9	1.61	1.5	2.05	4.6
81- 90	3.14	3.4	.15		2.5	4.64	4.9	2.95	8.1	3.7	9.85	6.6	2.9	2.82	2.7	3.62	5.0
91-100	2.65	3.42	.15	.5	1.6	5.25	3.4	2.12	7.7	1.36	7.5	10.35	5.8	4.55	2.05	3.25	5.07
101-110	2.3	2.72	.35	.5	3.4	3.55	3.6	2.78	7.75	2.89	3.65	7.75	2.38	5.83	1.3	6.46	5.95
111-120	1.57	3.00	.5		3.35	3.4	2.96	1.77	3.48	.81	10.4	6.9	5.2	4.6	1.25	5.45	4.05
121-130	1.75	2.45		.8	.75	3.00	1.97	.66	2.25	.81	5.9	8.6	1.87	3.3	2.03	.465	3.05
131-140	.5	.4			.9	1.2	1.57	.15	.30	.51	3.05	7.95	.30	1.06	1.6	2.1	1.25
141-150		.5			.35	1.2	.35			.81	.55	7.00	1.32	1.21	.5	3.0	1.2
151-160		.5			.45			.40	.30		.55	.85	.15	.35	.2	.07	.5
161-170		.35			.65	.35	.35				2.05	1.4		.51	.2	1.7	.4
171-180														.10			.2
Mean:—	1.82	1.63	.31	.42	1.45	2.23	2.13	1.53	3.73	1.57	4.31	4.96	2.21	2.08	1.16	2.99	2.47

INFORMATION ABOUT THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

WHAT ARE THE OFFICIAL PUBLICATIONS OF THE COUNCIL?

Each member of the Council receives the official journal, **The Mathematics Teacher,** the only magazine published that deals solely with the teaching of elementary and secondary mathematics. No progressive teacher of mathematics can afford to be without the help of this journal.

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Second Yearbook (1927)—Curriculum Problems in Teaching Mathematics. Price $1.50, 297 pages, in cloth.

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The magazine and these Yearbooks furnish much helpful program material.

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PROBLEMS

Proposed by H. L. Smith: Show that the inequality

$$|(x_1-x_2)\ (y_1+y_2)+(x_2-x_3)\ (y_2+y_3)+(x_3-x_1)\ (y_3+y_1)|$$
$$\leqq|x_1-x_2|\ |y_1-y_2|+|x_2-x_3|\ |y_2-y_3|+|x_3-x_1|\ |y_3-y_1|$$

holds for all real values of the six letters involved, and give a geometric interpretation.

Proposed by G. M. White. Given: A circle whose diameter is d, and whose center is o.

Another circle drawn, inside the first, whose diameter is d′ and whose center is o′,

A line is drawn from any outside point C, cutting the two circles at a and m respectively.

A line mb is drawn making angle Cmo′=angle o′mb.

Coo′ is a straight line.

Required: The distance oo′ so that the sum of the lines am and mb will be a constant.

Note: This is a problem of light, mn is a convex mirror and the line Cmb is a beam of light originating at C.

If this problem has no solution as stated, can it be solved with the line Co given?

A recent note from President Maizlish requests us to announce again that the Louisiana Academy of Sciences will hold its next annual meeting at Lafayette, La., in conjunction with the Louisiana-Mississippi M. A. of A. meeting at the same place.

AN APPEAL

The editors of the Mathematics News Letter earnestly urge all Louisiana and Mississippi mathematics teachers to become subscribers to the News Letter at once! Those whose subscriptions have expired are urged to renew them promptly.

This appeal is made **because:**

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(3) It is the only journal in America that undertakes to represent the **consolidated** interests of high school and college mathematics teachers.

(3) The News Letter will be used to bring to our teachers **the most recent tested-out advances in teaching projects and mathematical course materials.**

(4) Some of the contributors to its pages are classed among the **best teachers and scholars of the country.**

(5) **It should be supported by southerners because it is a southern institution.** This sentiment controlled when a teacher in far-away South Carolina recently wrote us: "It is indeed interesting to learn that such a publication is coming from our southern states. Enclosed is my check for a year's subscription."

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